Vale lo sviluppo

$$(1+x)^{\alpha} = \sum_{h=0}^{+\infty} \binom{\alpha}{h} x^h = 1 + \alpha x + \frac{\alpha(\alpha-1)}{2!} x^2 + \dots \quad \text{con} \quad \binom{\alpha}{h} = \frac{\alpha(\alpha-1)\cdots(\alpha-h)}{h!}$$

e quindi $\frac{1}{1-x} = \sum_{h=0}^{\infty} x^h$ Dunque $(1+x)^{\alpha} \simeq 1+\alpha x$

$$(1-x)^{-1} \simeq 1 + x$$

Piccole def.

(2.18) $\delta_{u^0} = \left(1 + 2\underline{\underline{E}}\, u_* \cdot u_*\right)^{\frac{1}{2}} - 1 \cong \cancel{1} + \frac{1}{\cancel{2}}\left(\cancel{2}\underline{E} u_* \cdot u_*\right) - \cancel{1} = E u_* \cdot u_*$

(2.2e) $\Rightarrow \cos \vartheta_{ij} = \frac{C_{ij}}{\sqrt{C_{ii}}\sqrt{C_{jj}}} = \frac{2E_{ij}}{\sqrt{1+2E_{ii}}\sqrt{1+2E_{jj}}} \simeq 2E_{ij}\left(1 - \frac{1}{2}2E_{ii}\right)\left(1 - \frac{1}{2}2E_{jj}\right) =$

$$= 2E_{ij}(1-E_{ii})(1-E_{jj}) \simeq 2E_{ij}$$

$$\to = -\sin \gamma_{ij} \cong -\gamma_{ij} + \frac{\gamma_{ij}^3}{3!} \Rightarrow \gamma_{ij} \cong -2E_{ij}$$

$C = I + 2E \Rightarrow$

$C^{-1} = (I+2E)^{-1} \cong I - 2E$

Usiamo l'espressione $\det(A - \lambda I) = -\lambda^3 + \lambda^2 I_1 - \lambda I_2 + I_3$ con $\lambda = -1$

$\Rightarrow \det(I + A) = 1 + I_1 + I_2 + I_3 \cong 1 + I_1 = 1 + \mathrm{tr} A$

(2.35) $\det C = \det(I + 2E) \simeq 1 + \mathrm{tr}(2E) = 1 + 2\,\mathrm{tr} E$

$$J = (\det C)^{\frac{1}{2}} \cong (1 + 2\,\mathrm{tr} E)^{\frac{1}{2}} \simeq 1 + \mathrm{tr} E$$

della Isc Mitsuhiro Kaneda ha citato

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(2.28) $\delta_{\Sigma} = J\sqrt{C^{-1} n_* \cdot n_*} - 1 \simeq (1 + \mathrm{tr} E)\sqrt{(I - 2E) n_* \cdot n_*} - 1 \simeq$

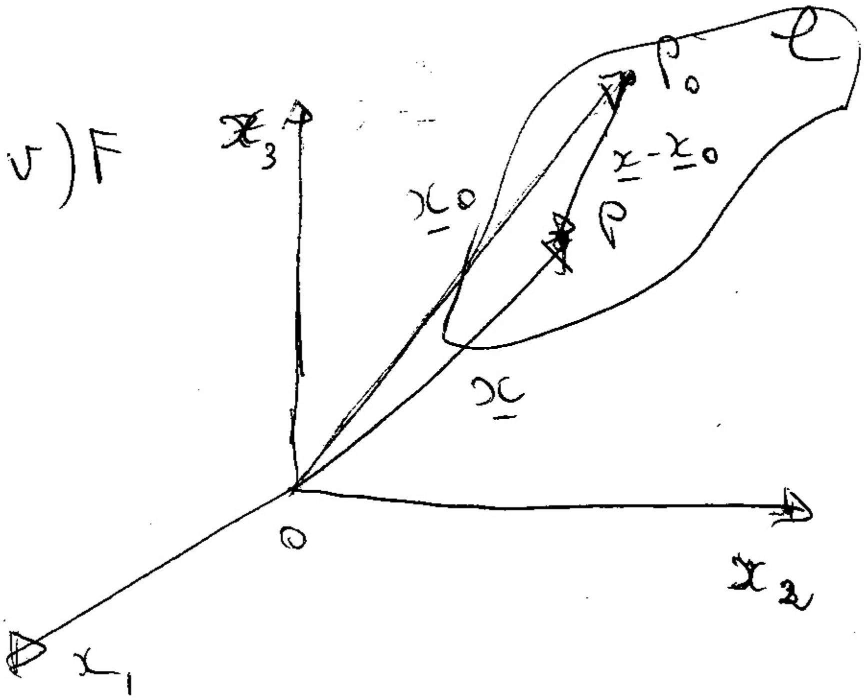
$$= (1 + \mathrm{tr} E)\sqrt{1 - 2E n_* \cdot n_*} - 1 = (1 + \mathrm{tr} E)\left(1 - \frac{1}{\cancel{2}}\cancel{2} E n_* \cdot n_*\right) - 1 \simeq$$

$$= \cancel{1} + \mathrm{tr} E - E n_* \cdot n_* - \cancel{1} \cancel{+} \mathrm{tr} E\,(E n_* \cdot n_*) \simeq \mathrm{tr} E - E n_* \cdot n_*$$

$$\underline{a} = \frac{\partial^2 x}{\partial t^2}(X,t) \qquad X \in \mathcal{C}^*$$

$$\underline{a} = \frac{d\underline{v}}{dt}(x,t) = \frac{\partial v}{\partial t} + \left(\operatorname{grad} \underline{v}\right)\underline{v}, \quad x \in \mathcal{C}$$

$$\dot{\underline{\underline{F}}} = \operatorname{Grad} \underline{v} = (\nabla v)F$$



$$[\omega \wedge (x - x_0)]_i =$$
$$= \varepsilon_{ijk}\, \omega_j\, (x - x_0)_k$$

$$(\nabla v)_{ij}$$
$$\parallel$$
$$\frac{\partial}{\partial x_l}\left[\varepsilon_{ijk}\, \omega_j (x - x_0)_k\right] = \varepsilon_{ijk}\, \omega_j \frac{\partial}{\partial x_l}(x_k - x_{0k}) =$$

$$= \varepsilon_{ijk}\, \omega_j\, \delta_{kl} = \underline{\varepsilon_{ijl}\, \omega_j} = -\varepsilon_{ilj}\, \omega_j = (-\underline{\underline{W}})_{il}$$

Esr. 2.1

$$\begin{cases} x_1 = kX_1 + X_2 + X_3 \\ x_2 = X_1 + kX_2 + X_3 \\ x_3 = X_1 + X_2 + kX_3 \end{cases} \qquad k \in \mathbb{R}, \quad \underline{\underline{F}} = \begin{pmatrix} k & 1 & 1 \\ 1 & k & 1 \\ 1 & 1 & k \end{pmatrix}$$

$$J = k^3 + 1 + 1 - k - k - k = k^3 - 3k + 2 = (k+2)(k-1)^2 > 0$$

$$= (k+2)(k^2 - 2k + 1) = k^3 \cancel{-2k^2} \cancel{+2k^2} - 4k + k + 2 \qquad \Big| \qquad \underline{k \neq 1 \text{ e } k > -2}$$

Esercizio 2.2.

$$F^{-1} = J^{-1} F^{cT} = \frac{-1}{11} \begin{pmatrix} -3 & -2 & -1 \\ -2 & -5 & 3 \\ -7 & -1 & +5 \end{pmatrix}$$

$$\begin{pmatrix} X_1 \\ X_2 \\ X_3 \end{pmatrix} = \underline{X} = \underline{\underline{F}}^{-1} \underline{x} = \frac{1}{11} \begin{pmatrix} 3x_1 + 2x_2 + x_3 \\ 2x_1 + 5x_2 - 3x_3 \\ 7x_1 + x_2 - 5x_3 \end{pmatrix}$$

Eser. 2.8.

$$F = \begin{pmatrix} 3 & -1 & 0 \\ -1 & 3 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad C = F^T F = F^2 = \begin{pmatrix} 10 & -6 & 0 \\ -6 & 10 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$0 = |C - \lambda I| = \begin{vmatrix} (10-\lambda) & -6 & 0 \\ -6 & (10-\lambda) & 0 \\ 0 & 0 & (1-\lambda) \end{vmatrix} = (1-\lambda)\left[(10-\lambda)^2 - 36\right] =$$

$$= (1-\lambda)\left(100 - 20\lambda + \lambda^2 - 36\right) = (1-\lambda)\left(\lambda^2 - 20\lambda + 64\right)$$

$$\lambda_1 = 4, \quad \lambda_2 = 16, \quad \lambda_3 = 1, \quad C' = \begin{pmatrix} 4 & 0 & 0 \\ 0 & 16 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$U' = \begin{pmatrix} 2 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$Cu = F^2 u = F(Fu) = F(\bar\lambda u) = \bar\lambda(Fu) = \bar\lambda^2 u$$

$$Cu = \lambda u \qquad \begin{pmatrix} 10 & -6 & 0 \\ -6 & 10 & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} u_1 \\ u_2 \\ u_3 \end{pmatrix} = \boxed{\begin{pmatrix} \lambda u_1 \\ \lambda u_2 \\ \lambda u_3 \end{pmatrix} = \begin{pmatrix} 10u_1 - 6u_2 \\ -6u_1 + 10u_2 \\ u_3 \end{pmatrix}}$$

(3) $\lambda_3 = 1$, $\hat{f}_3 = \hat{e}_3 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$

(1) $\lambda_1 = 4$ $\begin{cases} 10u_1 - 6u_2 = 4u_1 \\ -6u_1 + 10u_2 = 4u_2 \\ u_3 = 4u_3 \end{cases}$ $\begin{cases} u_1 = u_2 \\ u_2 = u_1 \\ u_3 = 0 \end{cases}$ $\underline{f}^2 = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$, $\hat{\underline{f}}_1 = \frac{\sqrt{2}}{2}\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$

(2) $\lambda_2 = 16$, $\hat{\underline{f}}_2 = \frac{\sqrt{2}}{2}\begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix}$, $Q^T = \begin{pmatrix} \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 \\ -\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} \hat{f}_1^T \\ \hat{f}_2^T \\ \hat{f}_3^T \end{pmatrix}$

$u_2 = -u_1$

$u_3 = 0$

$$Q^T C Q = C' \Rightarrow U = Q U' Q^T \qquad \det U = \det F = 8$$

$$R = F U^{-1} = \frac{1}{8} F U^{cT} = 8^{-1} F U^c$$

$$\underline{\underline{U}} = \begin{pmatrix} 3 & -1 & 0 \\ -1 & 3 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \underline{\underline{F}} \Rightarrow \underline{\underline{R}} = \underline{\underline{I}}$$

Esercizio 2.10:

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$$\underline{u} = \underline{x} - \underline{X} \quad , \quad \underline{x} = \underline{u} + \underline{X} = \begin{pmatrix} X_1 + k X_2^2 \\ X_2 \\ X_3 \end{pmatrix}$$

$$\boxed{\underline{\underline{F}} = \begin{pmatrix} 1 & 2kX_2 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}} \qquad J = 1$$

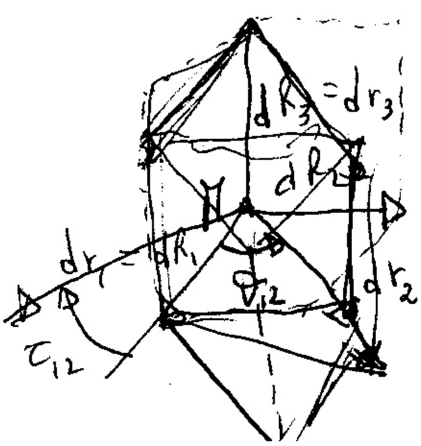
$$C = F^T F =$$

$$= \begin{pmatrix} 1 & 0 & 0 \\ 2kX_2 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 2kX_2 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 2kX_2 & 0 \\ 2kX_2 & (4k^2X_2^2+1) & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$d\underline{x} = \underline{\underline{F}}\, d\underline{X} \quad , \quad P^* = \Pi = (1,1,0)$$

$$d\underline{R}_1 = e_1\, dX_1 = \begin{pmatrix} dX_1 \\ 0 \\ 0 \end{pmatrix}, \quad d\underline{R}_2 = \begin{pmatrix} 0 \\ dX_2 \\ 0 \end{pmatrix}, \quad d\underline{R}_3 = \begin{pmatrix} 0 \\ 0 \\ dX_3 \end{pmatrix}$$

$$\underset{\underset{d\underline{R}_1}{\parallel}}{d\underline{r}_1} = \underline{\underline{F}}\, d\underline{R}_1 = \begin{pmatrix} dX_1 \\ 0 \\ 0 \end{pmatrix}, \quad d\underline{r}_2 = \begin{pmatrix} 2kX_2\, dX_2 \\ dX_2 \\ 0 \end{pmatrix}, \quad \underset{\underset{d\underline{R}_3}{\parallel}}{d\underline{r}_3} = \begin{pmatrix} 0 \\ 0 \\ dX_3 \end{pmatrix}$$



$$\delta_2 = \sqrt{c_{22}} - 1 = \sqrt{1 + 4k^2X_2^2} - 1$$

$$\cos\vartheta_{12} = \frac{2kX_2}{\sqrt{1 + 4k^2X_2^2}}$$